THE EFFECT OF DYNAMIC GEOMETRY SOFTWARE ON STUDENT MATHEMATICS TEACHERS' SPATIAL VISUALIZATION SKILLS

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ABSTRACT
Geometry is the study of shape and space. Without spatial ability, students cannot fully appreciate the natural world. Spatial ability is also very important for work in various fields such as computer graphics, engineering, architecture, and cartography. A number of studies have demonstrated that technology has an important potential to develop spatial skills. In the present study, the effect of dynamic geometry software (DGS) Cabri 3D on student mathematics teachers' spatial skills was examined. The study used a one-group pretest–intervention–posttest experimental design. In this context, 40 student mathematics teachers took "Purdue Spatial Visualization" (PSV) test as pretest at the beginning of the study, and then some implementations were carried out with these students by using Cabri 3D software for 8 weeks. The PSV test was retaken by students at the end of the study. That if there was a meaningful difference between the pretest and posttest results was examined by using paired sample t test. The findings demonstrated that these computer supported activities contributed to development student mathematics teachers' spatial skills.

Keywords: Spatial Ability, 3D Dynamic Geometry Software, Spatial Visualization

INTRODUCTION

The general objectives of geometry education can be summarized as: student should use geometry within the process of problem solving, understanding and explaining the physical world around them (Baki, 2001). The physical world around us cannot be explained by only two-dimensional Euclidean geometry. Because everything which we use, produce and buy has a three-dimensional geometric shape. Therefore, NCTM (2000) recommends that geometry instruction should include the study of three-dimensional geometry and provide students opportunities to use spatial abilities to solve problems.

Various authors including psychologists and educators have identified distinct components of spatial ability. According to McGee (1976), spatial ability consists of spatial skills as changing, rotating, bending and reversing of an object presented for stimulating in the mind. Linn and Petersen (1985) define spatial ability with mental processes being used in perceiving, storing, recalling, creating, arranging and making related spatial images. Spatial ability have identified several different spatial ability factors as understanding relations visually, making changes on shapes, rearrangement and interpreting them (Tartre, 1990). While some researchers examined spatial ability in two sub-dimension, spatial relations and spatial visualization (Burnett & Lane, 1980; Elliot & Smith, 1983; Pellegrino, Alderton, Shute, 1984; Clements & Battista, 1992), others examined it in three spatial factors (Thurstone, 1938): The ability to recognize the identity of an object when it is seen from different sights, the ability to imagine the movement or internal displacement among the parts of a configuration, the ability to think about those spatial relations in which the body orientation of the observer is an essential part of the problem. However, Maier (1996) examined spatial ability in five factors as spatial perception, visualization, mental rotation, spatial relations and spatial orientation. According to Maccoby and Jacklin, spatial ability consists of two important factors: Analytic and non-analytic factors. Analytic factor contains complex processes such as estimating close state of an object given as open. Non- analytic factor contains rotating an object (Maccoby and Jacklin, 1974). Olkun(2003), examined spatial ability in two major components; spatial visualization and spatial relations. Table 1 shows the definitions of the components, tests that can be used to measure related components, typical test items and their complexity (Olkun, 2003).

Table 1: Spatial Ability and its components

SPATIAL ABILITY		
Component	**Spatial Relations**	**Spatial Visualization**
Definition	Imagining the rotations of 2D and 3D objects as a whole body	Imagining the rotations of objects and their parts in 3D space in a holistic as well piece by piece fashion
Associated test	MGMP, Spatial Visualization Test, Primary Mental Abilities Test, French Reference Kit	MGMP, Spatial Visualization Test, Purdue Spatial Visualization Test, Minnesota Paper Form Board, Differential Aptitude Test, French

		Reference Kit
Typical test items	2D mental rotation, Cube comparison, 3D mental rotation.	Form board, Paper folding, Surface development, 2D-3D transformations
Complexity	Relatively simple tasks	Relatively complex tasks

As been in most of the studies, which were done about human mind, it is seen in the expressions above that it is not an easy task to define spatial ability and determine its factors. However, it may be said that spatial ability generally contains the skills such as rotating of an object, estimating its views from different aspects, changing its view according to the position of the spectator, estimating the folding of developments into three-dimensional object. Whatever its definition and content, it is clear that this skill has an important position in human thought. Because there is strong evidence to suggest that spatial ability plays an important role in the work of various fields such as computer graphics, engineering. In addition this, numerous studies have shown that spatial ability is positively related to problem solving ability as well as success in geometry and mathematics (Fennema & Sherman, 1977; Battista, Wheatley & Talsma, 1982, Battista, et al 1982; Fennema & Tartre, 1985, Moses, 1977).

A number of studies suggest that spatial skills can be developed through instruction (Bishop, 1980). For example, Ben Chaim et al (1989) found that spatial skills could be thought and thus learned by students. In his study of approximately 1000 middle grade students, scores on the Middle Grade Mathematics Project Spatial Visualization Test improved significantly after a three-week unit. Studies have also used computer environments to determine the effect of instruction on spatial skills. Travis and Lenon (1997) used MAPPLE – a computer software package with sophisticated graphing capabilities – in a pilot program developed use MAPPLE to enhance spatial skills. It was found that students in the experimental class scored better on the spatial skills test developed by the researchers. In contrast, some studies have not reported the same success. For example, Ferrini-Mundy (1987) found that an experimental group of calculus students drawn from a random sample of 334 students did not benefit from instruction that included spatial training modules.

The Dynamic Geometry Software Cabri 3D for exploring three-dimensional geometry was launched in 2004. It promises to revolutionize computer assisted visualization and reasoning in 3D geometry in much the same way as the earlier `dynamic geometry software' (DGS) has done for plane geometry. The expression of DGS is common name of the special geometry software such as Cabri Geometry, Geometers' Sketchpad and Cinderella. DGS provides an environment in which students can explore geometric relationships and make and test conjectures. One of the distinguishing features of a DGS is the ability to construct geometrical objects and specify relationships between them. Within the computer environment, geometrical objects created on the screen can be manipulated by means of the mouse (generally referred to as 'dragging'). What is particular to DGS is that when elements of a construction are dragged, all the geometric properties employed in constructing the figure are preserved (Jones, 1999). It can be said that these software have brought revolutionist innovations to geometry education and it is the most important step in geometry instruction since Euclid (De Villiers, 1996). It may be said that parallel to the reformist developments in plane geometry, the revolutionist developments have also existed in three-dimensional geometry especially after the Cabri 3D. Three-dimensional objects such as prism, cylinder and cone can be constructed, rotated and seen from a certain aspect on the screen and also prisms can be opened on the screen. Prisms and half plane can be intersected and thus, new three-dimensional objects may be formed. These features offer incredible opportunities to the student to develop their spatial skills. Also, because some measurements such as angle, length and surface area may be obtained on the screen via this software, the students have the opportunities to learn three-dimensional geometry by explorations. We may give the theorem about three perpendiculars as an example for it:

The Theorem About Three Perpendiculars: A straight line, that lies in a plane and is perpendicular to a projection of straight line inclined to this plane, is perpendicular to this straight line inclined to plane.

The students may explore the theorem themselves by following the steps as shown below using Cabri 3D. The students are asked to form a straight line on the plane and to take a point named A over this straight line. Then, they are asked to draw a perpendicular line from point A to this straight line on the plane. Then, they are asked to draw a perpendicular line from B point to the plane and to take a point named C on this perpendicular line. They can observe that the segment connecting the two perpendicular feet is vertical to the straight on the plane as seen in figure 1.

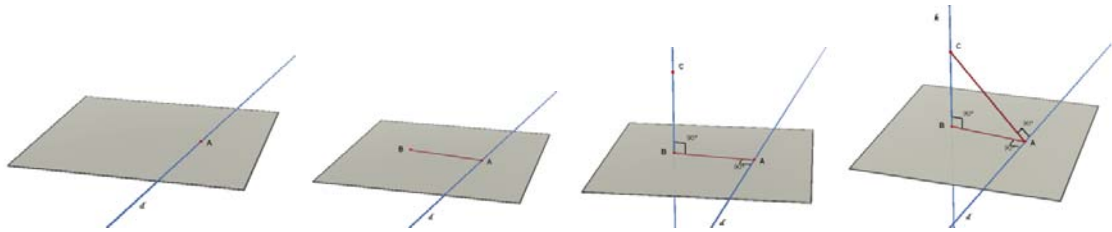

Figure 1. Screenshots of constructing the theorem about three perpendiculars

The Purpose of Study

Despite mathematics educators have optimistic ideas about Cabri 3D, we need more evidence to show possible effect of this software on student performance. Therefore, we should examine the effect of software on students' spatial skills.

The purpose of the present study is to determine the effect of the computer based activities designed by DGS Cabri 3D on the development of students mathematics teachers' spatial skills.

METHOD

Research Design

Pretest-intervention-posttest designs are uniquely appropriate for investigating the effects of educational innovations and are commonly used in educational research (Dugard & Todman, 1995). In this paper, it is not aimed to investigate the advantages or disadvantages of DGS environment rather than traditional environment. Therefore, true or quasi-experimental designs are not used in this study. So, in the present study, a one-group (no control) pretest-intervention-posttest experimental design was utilized to examine the effect of DGS Cabri 3D on the development of students mathematics teachers' spatial skills in Karadeniz Technical University.

Procedure and Intervention

At the beginning of the study, the students took PSV test as pretest. Then, various activities developed with Cabri 3D software were implemented at least 1.5 hours in a week along 8 weeks. During the treatment, the researchers were the teachers of the classroom. After 8 weeks, the students took the same test as posttest again. The schedule of the course activities is shown in table 1.

Table 1: The schedule of the course activities

Week	Course content
1st week	Learning about Cabri 3D Forming basic geometric objects (prisms, sphere, cylinder, cone etc)
2nd week	Obtaining point, circle, ellipse and hyperbole with the help of cone intersections
3rd week	Vertical projection and trigonometric relations
4th week	Reflection, transition and rotation conversions in three-dimensional environment
5th week	Forming various objects by cutting prisms on various surfaces, explanations by using the software and drawing them in their open states
6th week	Estimating close states of the objects given as open in Cabri and drawing the object by closing by the software
7th week	Making the objects intersected with surfaces and obtaining intersection curves
8th week	Free exercises

One of the activities students took during 6th week is seen below.

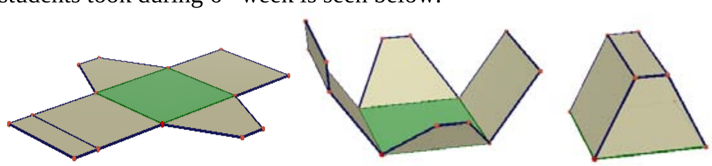

Figure 2. Screenshots of one of the activities step by step

Sample

This study took place during the fall of 2007 and consists of 40 student mathematics teachers with 22 male and

18 female in Karadeniz Technical University. The pretest and posttest (as the same test) implemented all the students but approximately 35-40 students attended the course weekly.

Instrument

In the study, PSV test was used as pretest and posttest for data collection. The test was developed by Roland Guay in 1976 and consists of 36 multiple-choice items in three sections (Developments, Rotations, and Views). Each section contains 12 questions. The explanation of each section of the test is seen below.

Developments: This section requires the student to study a pattern of three-dimensional objects and determine the correct answer from five possible shapes listed below it. A sample question from this section is seen below.

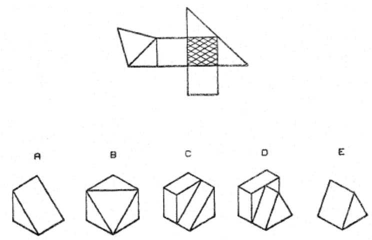

Figure 3: Sample question in Developments section of PSVT

Rotations: This section shows an object in two different positions. Shape one is rotated on the X, Y or Z axis to shape two, which is provided to show the rotation pattern. The student is required to select the object whose position represents the next rotation in the pattern. A sample question from this section is seen below.

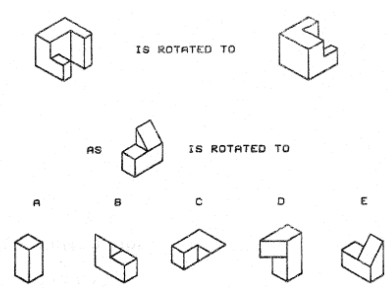

Figure 4: The sample question in Rotations section of PSVT

Views: This section tests a student's ability to visualize a three-dimensional object from various perspectives. In this section, an object is placed in a cube and one of the corners of the cube is marked. The student is asked to imagine that he looks at the object in a way that the marked corner will be between the object and the eye and then, to guess the view of the object. A sample question from this chapter is seen below.

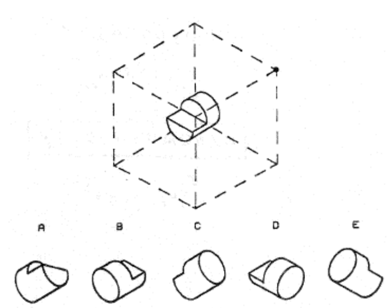

Figure 5: The sample question of the views section of PSVT

The PSVT was shown to be a valid and reliable instrument in different studies. Branoff (1998) calculated the internal consistency coefficients of .82 and .80. Battista, Wheatley and Talsma (1982) administered the PSVT to 82 pre-service teachers enrolled in an undergraduate geometry course and reported a KR-20 of .80. Guay (1980) used the PSVT on 217 university students, 51 skilled machinist, and 101 university students on three different occasions and reported an internal consistency coefficients (KR-20) of .87, .89 and .92.

The same test was employed as both pretest and posttest, may be seen an important problem for the study. However, Bertoline and Miller (1990) recommended that this test can be used as pretest and posttest to determine the spatial skills.

Data Analysis

Two types of analyses were conducted. First, descriptive statistics were employed to take a general photograph of students' spatial skills before and after the intervention. Thus, the researchers had an overall view about the effectiveness of the implementations. Second, in order to determine the effects of the treatment on the student mathematics teachers' spatial skills, Paired samples t test was used to compare the pretest and posttest scores.

RESULTS

In this chapter, student mathematics teachers' PSVT results are presented to examine the changes in their spatial skills before and after the intervention.

Descriptive statistic of the data obtained from PSVT before and after the intervention is seen in Table 2.

Table 2: Descriptive statistics of the students' PSVT scores before and after the intervention

	Before Intervention				**After Intervention**			
	Min.	Max.	Mean	SD	Min.	Max.	Mean	SD
Developments	2	10	6,2	2,0	2	12	7,9	2,9
Rotations	2	10	5,7	2,1	2	12	7,8	2,8
Views	0	7	3,8	1,4	1	10	5,7	2,0
Total	8	25	15,7	4,2	7	34	21,4	6,4

Figure 6 displays the results of the average scores of pre and posttest

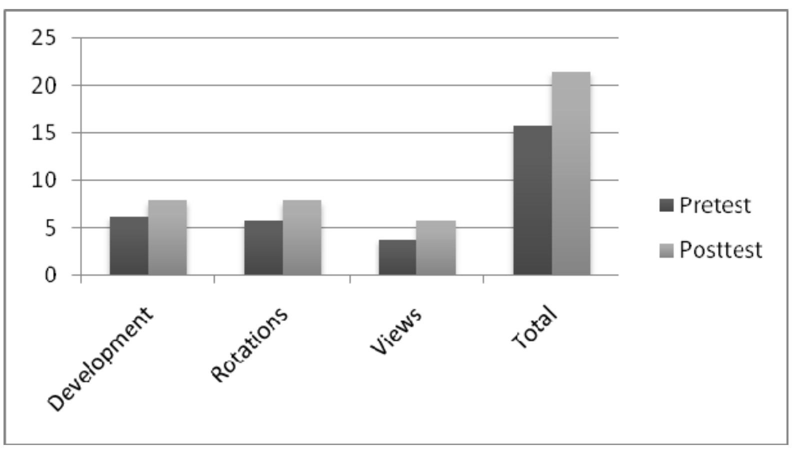

Figure 6: Average scores of pre and posttest

As seen on the average scores on Table 2 and Figure 6, the students gave the least correct answers in "Views" section and the most correct answers in "Developments" section before the intervention. Similar results were obtained in posttest, too.

As seen easily on the Figure 6, an increase in students' correct answer averages at each section of the test was emerged. From the Table 2, the increases in average scores are 1,7 in Developments section, 2,1 in Rotations section and 1,9 in Views section. Although the increases are close to each other, the highest one occurred in Rotations section. Average increase in the whole of the test is 5,7. These data demonstrate that the course caused an increase in the number of student' correct answers.

Paired sample t-test was applied to the data obtained from the whole of the test and its sections at a significant level of .05 to determine if this difference occurred in averages of the students' scores has a statistical meaning or not. Table 3 summarizes the results of the paired sample t-test analysis performed on the pretest and posttest.

Table 3: Mean Difference between Pre- and Post-tests

		N	df	t	p
1st pair	Developments	40	39	4,479	0,000
2nd pair	Rotations			4,910	0,000
3rd pair	Views			6,493	0,000
4th pair	Total			7,709	0,000

This evaluation suggested that as a result of the intervention program, there was a significant difference on student' spatial skills. This difference has occurred not only whole of the test results but also each section of the test results ($p < .05$ for all).

This result shows intervention designed by DGS Cabri 3D has a positive effect on spatial skills.

DISCUSSION

The present study aimed to determine whether the three-dimensional computer supported activities designed by DGS Cabri 3D have effects on developing student mathematics teachers' spatial skills or not. The results of PSVT students took at the beginning of the course showed that the students' spatial skills are quite low. Especially the average of 12 questions in Views section, 3.8 and in general the average of 36 questions, 15.7, shows the insufficiency of students' spatial skills. It is surprising that although the students learn three-dimensional objects and their features in early stages of elementary school and study three-dimensional objects in various lessons in Turkey, the averages are low. Because the research carried with student mathematics teachers, the researchers assumed that students' spatial skills might be high at the beginning of the study. However, it is an interesting point to see that students' spatial skills are low. Many factors may cause this unsuccessful result. We can summarize the most important two of them as below:

- One of the reasons of low scores is presenting three-dimensional spatial information in a 2 dimensional format on the blackboard in traditional geometry lessons in Turkey. Because of this limitation students don't have opportunities to create and manipulate 3D models that have vital importance of developing spatial skills. Cabri 3D has a great potential to remove this limitation. We can illustrate this as following blackboard drawing and Cabri 3D diagram.

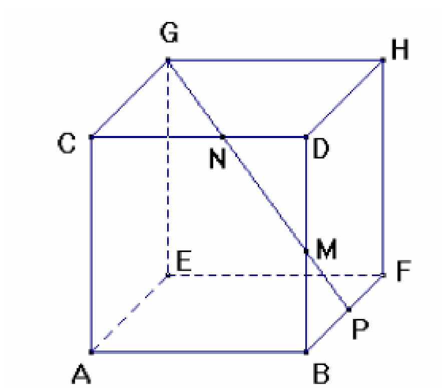

Figure 7. An example of blackboard drawing (static)

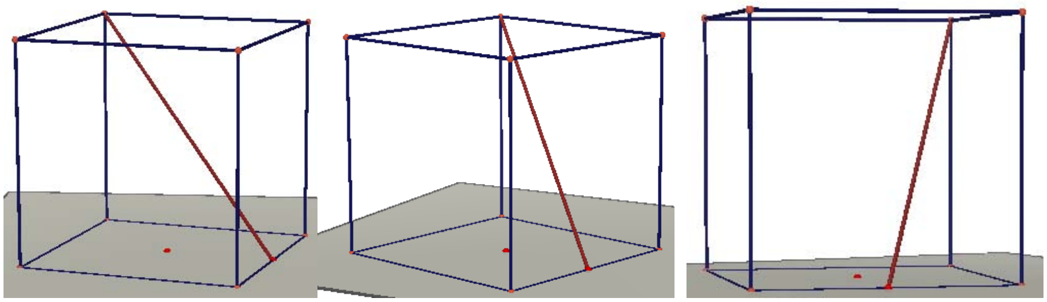

Figure 8. A Cabri 3D diagram (Students easily rotate the cube dynamically)

- Because of university entrance exam with multiple choose questions, geometry teaching in Turkey is largely based on procedural teaching. For example, surface area and volume formulas of basic 3 dimensional geometric objects, frequently asked in this entrance exam, are learned parrot fashion by Turkish students. But this cannot develop students' spatial skills. For changing this negative result, we should enrich the education programs with various spatial skills activities.

The ability of individuals to visualize and manipulate mental images has been recognized as an important cognitive ability in both mundane activities and academic endeavors. Therefore, students' spatial skills should be generally determined and the other reasons of the fail should be examined in Turkey with comprehensive researches.

After the intervention, there has been a development on the students' spatial skills. When the results are examined, it is seen that the most increase on students' scores is in the Rotations section. Beside other features, the characteristic of DGS Cabri 3D which allows rotating 3 dimensional geometric object on the screen by movements of mouse made the increase in Rotations be the highest level.

The dynamic nature of DGS provides students to learn geometric concepts and to explore geometric relationships easily. And also, it is a clear conclusion from this study that DGS especially Cabri 3D assist students to develop their spatial skills. Therefore, it can be recommended to mathematics teachers to use DGS Cabri 3D for developing their students' spatial skills.

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